Open Problems
in Computability Theory and Descriptive Set Theory
(Tentative Version)

June 24, 2025

Foreword

These open problems were presented in the Problem Sessions held during the Tianyuan Workshop on Computability Theory and Descriptive Set Theory, June 16-20, 2025. The problems are organized into sections named after their posers. Notes are taken and the list is compiled by Wei Dai, Feng Li, Ruiwen Li, Ming Xiao, Xu Wang, Víctor Hugo Yañez Salazar, and Yang Zheng.

Contents

Theodore Slaman

Given $A \subseteq \mathbb{R}$. Define

$$I(A) = \{f \text{ gauge function } : H^f(A) > 0\}.$$

Problem 1. Natural questions about $A \mapsto I(A)$:

1. Does the range of I have the same cardinality as $2^{\mathbb{R}}$? (Seems yes under GCH.)
2. If consider the closed set C, $I(C)$ is arithmetic ($\mathbf{\Sigma}^0_2$). What's the complexity of the set

   $$\{\mathbf{\Sigma}^0_2 \text{ formula } \varphi\colon \varphi \text{ defines an } I(C) \text{ for some } C\}?$$

   Is it $\mathbf{\Sigma}^1_2$-complete? (It is $\mathbf{\Pi}^1_1$-hard.)
3. Is the answer to "Borel hierarchy is propert wr to the range of I" independent of ZFC?
4. Is it consistent with ZFC that for every f and $A \subseteq \mathbb{R}$, if $H^f(A) > 0$, then there is a subset A_0 such that $H^f(A_0)$ is finite and positive?

References

[1] C. A. Rogers and K. Falconer, Hausdorff Measures, 2nd ed. Cambridge Mathematical Library. Cambridge University Press, Cambridge, 1998.

[2] K. Falconer, Fractal Geometry: Mathematical Foundations and Applications. John Wiley & Sons, Ltd, 2003.S. D. Iliadis.

[3] L. Crone, L. Fishman, and S. Jackson, Equivalence relations and determinacy. *Journal of Mathematical Logic* 22 (2022), Article 2250003.

George Barmpalias

Problem 2. Let $L_x = \{z : z \leq_K^+ x\}$ and $U_x = \{z : z \geq_K x\}$. Are the following true?

(1) L_x is countable iff $\liminf_{n\to\infty}(K(x \upharpoonright n) - K(n)) < \infty$;

(2) U_x is countable iff $\liminf_{n\to\infty}(K(n) + n - K(x \upharpoonright n)) < \infty$.

Problem 3. Are the following true?

(1) For any c.e. set A, there exists a c.e. set $D \subseteq 2\mathbb{N}$, such that $A \equiv_{rK} D$ and $C(A \upharpoonright n|D \upharpoonright n) = C(D \upharpoonright n|A \upharpoonright n) = O(1)$;

(2) For all x, there is a random z such that $z \geq_{rK} x$ (or $z \geq_K x$).

Consider the Even Number Game defined in [1]. Given a natural number $k \geq 2$, the game G_k is a game with two players who take turns to play natural numbers. Player 1 plays a set A of k integers where none of the members of A has been played by her before; then Player 2 plays an even number between $\min(A)$ and $\max(A)$ (inclusive) which has not been played by him before. Player II loses when he has no legal moves.

It was shown in [1] that for $k = 2.3$, Player 1 has a winning strategy in the game G_k. For $k \geq 4$, this is unknown.

Problem 4. For $k \geq 4$, does Player 1 have a winning strategy in the game G_k?

References

[1] G. Barmpalias, X. Zhang, and B. Zhan, Compression of enumerations and gain. *arXiv:2304.03030* (2023).

Jun Le Goh

Recall that **WF** is the collection of all well-founded trees on ω and **UB** is the collection of all trees on ω which have a unique infinite branch. It is known that **WF** and **UB** are both coanalytic complete.

Saint Raymond [1] proved in 2007 that (**WF**, **UB**) is a coanalytic complete pair using pure Descriptive Set Theory methods, i.e., for every disjoint pair (A, B) of coanalytic sets in the Baire space ω^ω, there is a continuous function $f\colon \omega^\omega \to \omega^\omega$ such that $A = f^{-1}[\mathbf{WF}]$ and $B = f^{-1}[\mathbf{UB}]$.

Using recursion theory, Goh proved that any coanalytic separator of **WF** and **UB** is coanalytic complete. The method also proves Saint Raymond's theorem.

Problem 5. Can this be proved using Descriptive Set Theoretic methods?

References

[1] J. Saint Raymond, Complete pairs of coanalytic sets. *Fundamenta Mathematicae* 194 (2007), 267-281.

Liang Yu

Problem 6 (Downey–Hirschfeldt–Miller–Nies)**.** Is there an x such that $\Omega^x \geq_T 0''$ where

$$\Omega^x = \sum_{\sigma \in 2^{<\omega}} 2^{-K^x(\sigma)}$$

and

$$K^x(\sigma) = \min\{|\tau| \colon U^x(\tau) = \sigma\}?$$

Related to this question, Yu and Zhao proved the following proposition:

Proposition 7 (Yu–Zhao)**.** $\forall y \exists x\ \Omega^x \oplus 0' \geq_T y$.

Theorem 8 (Velickovic–Woodin [1])**.** If A is $\boldsymbol{\Sigma}^1_1$ and $\sup\{\omega_1^{CK(x)} \colon x \in A\} = \omega_1$, then $\forall \gamma \exists x_0, x_1, x_2, x_3 \in A$,

$$\gamma \leq_h x_0 \oplus x_1 \oplus x_2 \oplus x_3.$$

Problem 9. (a) Is there a recursion theoretical proof for the above theorem of Velickovic–Woodin?

(b) Is the following true: For any $\Sigma^1_1(y)$ set A, if there is $x \in A$ such that $\omega_1^{CK(x)} > \omega_1^{CK(y)}$, then the Velickovic–Woodin theorem holds for A?

References

[1] B. Velickovic and W. H. Woodin, Complexity of reals in inner models of set theory. *Annals of Pure and Applied Logic* 92 (1998), 283-295.

Wei Wang

We start with some definitions.

A tree $T \subseteq 2^{<\omega}$ is *positive* if $[T]$ has measure > 0.

Consider the following statements:

> P^+: For every positive tree T, there is subtree $S \subseteq T$ which is perfect and positve.
>
> P: For every positive tree T, there is subtree $S \subseteq T$ which is perfect
>
> P^-: For every positive tree there are $(X_n : n \in \omega)$ such that $X_n \in [T]$.

It is known that

$$\mathrm{WKL}_0 \vdash P^+$$

and

$$\mathrm{RCA}_0 \vdash P^+ \to P \to P^- \to \text{1-RAN} \leftrightarrow \mathrm{WWKL}_0,$$

where 1-RAN means there exists a 1-ramdom. But we also know that

$$P^+ \nvdash \mathrm{WKL}_0,$$

$$\mathrm{RCA}_0 \nvdash P \to P^+$$

and

$$\mathrm{RCA}_0 \nvdash \text{1-RAN} \to P^-.$$

The questions are:

Problem 10. Is it true that $\mathrm{RCA}_0 \vdash P^- \to P$?

Problem 11. How about using dimension? (Define the above concepts by replacing measure by dimension.)

Consider the following statements:

> $\mathrm{PHP}(\Sigma_{n+1})$: There is no injective $F \in \Sigma_{n+1}$ such that for some positive x, $F : x+1 \to x$.
>
> $\mathrm{WPHP}(\Sigma_{n+1})$: There is no injective $F \in \Sigma_{n+1}$ such that for some positive x, $F : 2x \to x$.
>
> $\mathrm{GPHP}(\Sigma_{n+1})$: For any x, there is a y such that there exists no Σ_{n+1} injection with $F : y \to x$.
>
> $\mathrm{GARD}(\Sigma_{n+1})$: For any x, there is no Σ_{n+1} injection F with $F : \mathbb{N} \to x$.

It is known that, over $\mathrm{I}\Sigma_n$,

$$\mathrm{B}\Sigma_{n+1} \leftrightarrow \mathrm{PHP}(\Sigma_{n+1}) \vdash \mathrm{WPHP}(\Sigma_{n+1}) \to \mathrm{GPHP}(\Sigma_{n+1}) \to \mathrm{CARD}(\Sigma_{n+1}).$$

But we also know that

$$\mathrm{PHP}(\Sigma_{n+1}) \nvdash \mathrm{GPHP}(\Sigma_{n+1}) \to \mathrm{WPHP}(\Sigma_{n+1})$$

and

$$\mathrm{PHP}(\Sigma_{n+1}) \nvdash \mathrm{CARD}(\Sigma_{n+1}) \to \mathrm{GPHP}(\Sigma_{n+1}).$$

Finally, consider

> $\mathrm{FRT}^e_k(\Sigma_{n+1})$: For any x, there is a y such that every Σ_{n+1} $C : [y]^e \to k$ has a homogeneous set H with $|H| \geq x$.

It is known that

$$\mathrm{I}\Sigma_n + \mathrm{WPHP}(\Sigma_{n+1}) + \mathrm{FRT}^e_k(\Sigma_{n+1})(e, k \text{ standard}) \nvdash \mathrm{B}\Sigma_{n+1}$$

and

$$\mathrm{FRT}^e_k(\Sigma_{n+1}) \nvdash \mathrm{WPHP}(\Sigma_{n+1}).$$

The questions are:

Problem 12. $\mathrm{I}\Sigma_{n+1} + \mathrm{WPHP}(\Sigma_{n+1}) \vdash \mathrm{FRT}^2_2(\Sigma_{n+1})$? $\mathrm{FRT}^2_2(\Sigma_{n+1}) \vdash \mathrm{FRT}^3_2(\Sigma_{n+1})$?

Problem 13. The first order theory 2-RAN is between $\mathrm{CARD}(\Sigma_2)$ and $\mathrm{WPHP}(\Sigma_2)$, but how about $\mathrm{GPHP}(\Sigma_2)$ and 2-RAN?

Andre Nies

Profinite groups

A f.g. group $S = F_k/N$ is effectively residually finite (e.r.f.) if there is an algorithm that, on input $w \in F_k$, in case $w \notin N$ computes a finite group Q and homomorphism $r : F_k \to Q$ such that $r(N) = \{e\}$ and $r(w) \neq e$.

Theorem 14. A f.g. group L is isomorphic to a subgroup of some computable profinite group that is generated by finitely many computable paths iff the following two conditions hold:

(a) L has a Π^0_1 word problem (call this a Π-group)

(b) L is effectively residually finite.

Problem 15. (a) Can such a group L have unsolvable word problem?

(b) Is there a f.g., residually finite Π-group that is not effectively r.f.?

(c) Morozov (Higman's question revisited, 2000) constructed a Π-group that is not isomorphic to a subgroup of S_{rec}. Can we make such a group r.f.? It can't be e.r.f.

For each left Σ_2 real $r \in [0, 1]$ there is a computable profinite group and computable subgroup with Hausdorff dimension r.

Problem 16. Which values can occur for r when G is also topologically finitely generated?

Nies, Segal and Tent 2021 [1] studied expressiveness of f.o. logic for profinite group. Many such groups are axiomatised by single sentence in the language of groups, among this reference class.

Problem 17. For a f.o. sentence φ, what is the possible complexity of $\{G : G \models \varphi\}$?

The following problem was also posed during the workshop, and by the end of the workshop Gao and Nies realized that the answer is yes.

Problem 18. Is there some Borel class of profinite groups with isomorphism relation properly between S_∞-complete and smooth? In particular, how about the Abelian case?

Update: The answer to this problem is yes. Due to the Pontryagin duality, the isomorphism relation of profinite abelian groups is Borel bireducible with the isomorphism of countable torsion abelian groups. It is well known that the latter relation is classified by the Ulm invariants, and is known to be $\mathbf{\Sigma}^1_1$-complete. In terms of Borel reducibility, it is strictly above the smooth equivalence relation and strictly below the graph isomorphism; in particular, it is not above E_0.

One could still consider other classes of profinite groups, such as small (only finitely many subgroups of each finite index), or strongly complete (each subgroup of finite index is open). See Dan Segal's work for references.

Complexity of isomorphism of oligomorphic groups

Problem 19. Nies, Schlicht and Tent proved in 2019 [2] that the topological isomorphism of oligomorphic groups is $\leq_B E_\infty$. Is it smooth?

Theorem 20 (Nies-Paolini [3])**.** (1) Let G be a non-archimedean Roelcke precompact group. Then the group $\mathrm{Aut}(G)$ of continuous automorphisms of G carries a natural Polish topology and $\mathrm{Inn}(G)$ is closed in it.

(2) Suppose in addition that G is oligomorphic, then $\mathrm{Out}(G) = \mathrm{Aut}(G)/\mathrm{Inn}(G)$ with the quotient topology is totally disconnected, locally compact (t.d.l.c.).

Problem 21. If G is oligomorphic, is $\mathrm{Out}(G)$ always profinite?

Problem 22. Is there a Borel, invariant class C of closed subgroups of S_∞ with isomorphism relation not Borel below graph isomorphism? Analytic complete? How about the class C of pro-countable groups?

Computability theory

In [4, Section 3.1] the authors showed the following implications

$$\text{1-generic } \Delta^0_2 \;\Rightarrow\; \text{index guessable} \;\Rightarrow\; \text{computes no maximal tower} \;\Rightarrow\; \text{low}$$

Problem 23. In the above diagram, are the first and the second implications reversible?

References

[1] A. Nies, D. Segal, K. Tent, Finite axiomatizability for profinite groups, *Proceedings of the London Mathematical Society* 123 (2021), no. 6, 597–635.

[2] A. Nies, P. Schlicht, K. Tent, Coarse groups, and the isomorphism for oligomorphic groups, *Journal of Mathematical Logic* (2021): 2150029.

[3] A. Nies, G. Paolini, Oligomorphic groups, their automorphism groups, and the complexity of their isomorphism, arXiv: 2410.02248.

[4] S. Lempp, J. S. Miller, A. Nies, M. I. Soskova, Maximal towers and ultrafilter bases in computability theory, *Journal of Symbolic Logic* 88 (2023), no. 3, 1170–1190.

Nikolay Bazhenov

Let x be a Turing degree. A computable structure S is x-computably categorical if for any computable structure $A \cong S$, there exists an x-computable isomorphism $f : A \to S$.

A Turing degree d is the degree of categoricity for S if d is the least degree such that S is d-computably categorical.

A computable structure S is decidable if its complete diagram is computable: i.e., given a first-order formula $\psi(\bar{x})$ and a tuple $\bar{a}$ from S, one can computably check whether $S \models \psi(\bar{a})$.

A decidable structure S is decidably x-categorical if for any decidable structure $A \cong S$, there exists an x-computable isomorphism $f : A \to S$.

A degree d is the degree of decidable categoricity for S if d is the least degree such that S is decidably d-categorical.

Problem 24. Every degree of decidable categoricity is a degree of categoricity. Is the converse true?

A conjecture for this problem is that it is true for $d \geq 0^{(\alpha)}$, where α is 'sufficiently large' computable ordinal. Say, for $\alpha = \omega$. What happens with d.c.e. degrees d?

Ceer stands for computably enumerable equivalence relation on ω. A diagonal function (or a fixed-point-free function) for a ceer E is a total function $g(x)$ satisfying $\neg g(x)Ex$ for all $x \in \omega$.

Problem 25. The following are three versions of this problem.

(i) Describe the class **Diag** containing those Turing degrees d such that every ceer $E \neq \mathrm{Id}_1$ admits a d-computable diagonal function. Here we have some partial results:

- (a) $\mathbf{PA} \subseteq \mathbf{Diag} \subseteq \mathbf{DNR}$ [Badaev, B., Kalmurzayev, Mustafa 2024].
- (b) There exists a Martin-Löf random x such that $x \notin \mathbf{Diag}$ [Ng].

(ii) What is the reverse-mathematical strength of the statement "Every Σ^0_1-definable equivalence relation $E \neq \mathrm{Id}_1$ has a diagonal function"?

(iii) What is the Weihrauch degree of the corresponding problem?

Keng Meng Selwyn Ng

A right-c.e. metric space (S, d) consists of a countable set $S = \{c_0, c_1, \dots\}$ and $d : \mathbb{N}^2 \to \mathbb{R}$ such that $d(c_i, c_j)$ is a right-c.e. real number, uniformly in i, j. Same for a left-c.e. metric space.

Problem 26. Does every (effectively) compact left-c.e. Polish space have a computable (right c.e.) Polish copy? Related to this problem, Melnikov and Ng proved that there is a left c.e. Polish space with no computable Polish presentation. Similarly, Koh, Melnikov and Ng proved that there is a left c.e. Polish space with no right c.e. Polish presentation. As a positive result, Melnikov and Ng proved that every left c.e. Stone space is homeomorphic to a computable Polish space.

A computable Polish space X is α-categorical if for every pair of computable metric spaces M, N such that $\bar{M} \cong \bar{N} \cong X$, there is an α-computable homeomorphism between $\bar{M}$ and $\bar{N}$. The least Turing degree α is the degree of (topological) categoricity of X. X is (topologically) computably categorical if it is 0-categorical.

Problem 27. Is there an infinite computable Polish space that is topologically computably categorical?

Problem 28. Does the Baire space have a degree of categoricity?

Problem 29. Is every/any c.e. degree the degree of topological categoricity?

Chong Chi Tat

Let M be a model satisfying $\mathrm{RCA}_0 + \mathrm{B}\Sigma_n + \neg\mathrm{I}\Sigma_n$. Fix a Π_n subset X of M and assume $\langle A_s : s \in M\rangle$ is a sequence of pairwise disjoint M-finite sets such that $\forall s\,(A_s \cap X \neq \emptyset)$.

Problem 30. Is there a definable in M choice function f of $\langle A_s\rangle$, i.e., $f(s) \in A_s \cap X$?

Tree version: Without loss of generality, let us state the problem for $n = 2$.

Problem 31. Consider a tree T which is $\Pi^0_1(\emptyset')$ in M. Assume there is a $g \in [T]$ such that $M[g] \models \mathrm{B}\Sigma_2$. Is there a definable path $h \in [T]$ such that $M[h] \models \mathrm{B}\Sigma_2$?

Su Gao

The graph complement problem

In 2021, Matt Foreman asked the following question:

Problem 32. Is the set $\{G \in 2^{\omega\times\omega} : G \text{ is a graph on } \omega \text{ and } G \cong G^c\}$ Borel?

Here $G^c = \{(m,n) \in \omega \times \omega : m \neq n \wedge (m,n) \notin G\}$ is the complement graph of G.

The problem was solved during the workshop.

Theorem 33 (Feng Li–Ruiwen Li–Ming Xiao; Riley Thornton)**.** The set

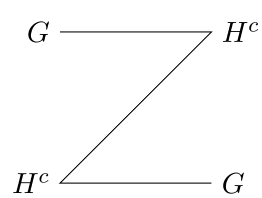
$$\{G \in 2^{\omega\times\omega} : G \text{ is a graph on } \omega \text{ and } G \cong G^c\}$$

is $\boldsymbol{\Sigma}^1_1$-complete.

Proof. We define a continuous function Φ from GRAPH^2 to GRAPH, such that

$$\begin{aligned}(m,n) \in \Phi(G,H) \iff & m = 4k_1, n = 4k_2 \text{ and } (k_1,k_2) \in G;\\ & m = 4k_1+1, n = 4k_2+1 \text{ and } (k_1,k_2) \in H^c;\\ & m = 4k_1+2, n = 4k_2+2 \text{ and } (k_1,k_2) \in H^c;\\ & m = 4k_1+3, n = 4k_2+3 \text{ and } (k_1,k_2) \in G;\\ & m = 4k_1, n = 4k_2+1;\\ & m = 4k_1+1, n = 4k_2+2;\\ & m = 4k_1+2, n = 4k_2+3;\end{aligned}$$

The following figure depicts our definition for Φ:



Now if $G \cong H$, it is easy to see $\Phi(G,H)$ is isomorphic to its complement.

Conversely, if $\Phi(G,H) \cong \Phi(G,H)^c$, let f be the witness of their isomorphism, and d_1, d_2 be the distance function on $\Phi(G,H)$ and $\Phi(G,H)^c$ respectively. Note that in $\Phi(G,H)$,

$4\mathbb{N} \cup 4\mathbb{N}+3 = \{x \in \omega : \exists y\, d_1(x,y) = 3\}$. This is similar for $\Phi(G,H)^c$. So f sends $4\mathbb{N} \cup 4\mathbb{N}+3$ to $4\mathbb{N}+1 \cup 4\mathbb{N}+2$. Also note that

$$\{x \in \omega : \exists y\,(d_1(x,y) = 3) \wedge d_1(x,0) \leq 2\}$$

is a copy of G,

$$\{x \in \omega : \exists y\,(d_2(x,y) = 3) \wedge d_2(x,f(0)) \leq 2\}$$

is a copy of H, and f maps the former set to the latter set. So $G \cong H$. □

Using similar methods, Feng Li and Ruiwen Li also obtained the following:

Theorem 34. The following sets are $\mathbf{\Sigma}^1_1$-complete:

1. The set of all countable directed graphs G where $G \cong G^c$;
2. The set of all countable tournaments G where $G \cong G^c$.

A directed graph G is a *tournament* if for any distinct vertices $x, y \in G$, exactly one of (x,y) and (y,x) is an edge in G.

Some hyperfiniteness problems

The following is the well known Union Problem in the theory of hyperfinite equivalence relations.

Problem 35 (The union problem)**.** If E is a countable Borel equivalence relation and $E = \cup_n F_n$, where F_n is hyperfinte and $F_n \subseteq F_{n+1}$ for each $n \in \omega$, is it true that E is hyperfinite?

The following theorem was recently proved.

Theorem 36 (Frisch–Shinko–Vidnyánszky [1])**.** If there is a counterexample to the union problem, then the set of all hyperfinite equivalence relations is $\mathbf{\Sigma}^1_2$-complete.

Analogous to the above, we call a countable Borel equivalence relation *hyperfinite-over-hyperfinite* (or shortly, *hf/hf*) if there is a Borel partial order $\leq$ on X such that

(i) if $x \leq y$, then xEy,

(ii) the order type of $\leq\restriction_{[x]}$ is a suborder of $\mathbb{Z}^2$.

Problem 37 (The hf/hf problem)**.** If E is hf/hf, is it hyperfinite?

There is an equivalent characterization for hyperfiniteness of hf/hf equivalence relation:

Theorem 38 (Gao–Xiao [2])**.** If E is hf/hf, then E is hyperfinite iff E admits a $\mathbb{Z}^2$ ordering which is self-compatible.

Inspired by the Frisch-Shinko-Vidnyánszky theorem, we ask:

Problem 39. Suppose there is a counterexample to the hf/hf problem. Is the set of all hyperfinite equivalence relations $\boldsymbol{\Sigma}^1_2$-complete?

References

[1] J. Frisch, F. Shinko, Z. Vinyánszky, Hyper-hyperfiniteness and complexity, preprint, 2024, arXiv: 2409.16445.

[2] S. Gao, M. Xiao, An order analysis of hyperfinite Borel equivalence relations, *Journal of Symbolic Logic*, published online, DOI: 10.1017/jsl.2024.26.

Jing Yu

Problem 40 (Weiss' question)**.** Is the orbit equivalence relation of Borel action of countable amenable group hyperfinite?

The orbit equivalence relations of actions of the following groups are proved to be hyperfinite:

$\mathbb{Z}$(Slaman–Steel),

$\mathbb{Z}^n$(Weiss),

Groups of polynomial growth(Jackson–Kechris–Louveau),

Abelian groups(Gao–Jackson),

Locally nilpotent groups(Seward–Schneider),

Polycyclic groups(Conley–Jackon–Marks–Seward–Tucker-Drob).

Also the connected equivalence relations of the following graphs are proved to be hyperfinite:

Graphs of polynomial growth(Bernshteyn–Yu),

Graph of growth less than $\exp(r^c)$ for some small enough $0 < c < 1$(Grebík–Marks–Rozhoň–Shinko).

Problem 41. How about graphs of uniform subexponential growth, i.e., of growth less than $\exp(r^c)$ for some $0 < c < 1$?

Riley Thornton

Let $\mathrm{Aut}([0,1],\lambda)$ denote the space of all automorphisms of the Lebesgue measure with the weak topology. Consider the action of F_∞ on $[0,1]$ by automorphisms of Lebesgue measure. Then the space of all actions, $\mathrm{Act}(F_\infty,([0,1],\lambda))$, is a closed subspace of $\mathrm{Aut}([0,1],\lambda)^{F_\infty}$.

Problem 42. $\{x \in \mathrm{Act}(F_\infty,([0,1],\lambda))\colon \mathrm{Sch}(x)$ has a measurable perfect matching$\}$ is $\mathbf{\Sigma}^1_1$-compete?

Problem 43 (Halmos, 1956)**.** What $T \in \mathrm{Aut}([0,1],\lambda)$ has $S \in \mathrm{Aut}([0,1],\lambda)$ with $T = S^2 = S \circ S$?

Conjecture: $\{T \in \mathrm{Aut}([0,1],\lambda)\colon \exists S \in \mathrm{Aut}([0,1],\lambda)\ T = S^2\}$ is $\mathbf{\Sigma}^1_1$-complete.

Wei Dai

Problem 44. If $\vec{G}$ is a Borel l.f. directed graph whose In-degree is uniformly bounded and Out-nbhd has polynomial growth, is the connected equivalence relation of $\vec{G}$ hyperfinite?

Problem 45. Let α be a free, p.m.p and ergodic action of $\mathbb{F}_2$ on measure space (X, μ).

(i) If $e \neq x \in \mathbb{F}_2$, is there a free, p.m.p action β such that $E_\alpha = E_\beta$ and x acts ergodically.

(ii) (Miller-Tserunyan). Is there a free, p.m.p action β such that every nontrivial element of $\mathbb{F}_2$ acts ergodically.

Jialiang He

A maximal eventually different family (MED) is a family $\varepsilon \subseteq \omega^\omega$ such that:

(i) $\forall f \neq g \in \varepsilon \; |f \cap g| < \infty$ and

(ii) $\forall f \in \omega^\omega \exists f \in \varepsilon \; |f \cap g| = \infty$.

Theorem 46. There exists a closed MED.

(Gao asked if there is a Π^0_1 MED.)

Let I be an ideal on ω. We can define I-MED anolog to the above definition. A family $\varepsilon \subseteq \omega^\omega$ is called an I-MED if:

(i) For all $f \neq g \in \varepsilon$, $\{n \colon f(n) = g(n)\} \in I$ and

(ii) $\forall h \in \omega^\omega \exists f \in \varepsilon \; \{n \colon f(n) = g(n)\} \in I^+$.

Theorem 47. If I is an F_σ ideal or $F = \mathrm{Fin}^\alpha$, $\alpha < \omega_1$, then there exists a closed I-MED.

Problem 48. For all Borel I, is there a closed I-MED?

Problem 49. For maximal ideal I, is there a closed I-MED?

For these problems, it is enough to find a Borel I-MED.

Theorem 50. For any ideal I, if there is a Borel I-MED, then there is a closed I-MED.